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245a. Proposed by W. J. GREENSTREET, A. M., Editor of The Mathematical Gazette, Stroud, England.

PCP' , DCD' are conjugate diameters of an ellipse; PN , DM are the ordinates to the major axis at P and D ; show $CM/PN = CN/DM = AC/BC$, and that AP and BD' are parallel, and that AP' is parallel to BD .

I. Solution by J. SCHEFFER, Kee Mar College, Hagerstown, Md.

Denote AC by a , BC by b , CP by a' , CD by b' , $\angle PCA$ by ϕ , and $\angle DCM$ by λ . From the well known relations of conjugate diameters of an ellipse, we have

$$\sin^2 \phi = \frac{b^2(a^2 - a_1'^2)}{a_1'^2(a^2 - b^2)}, \quad \cos^2 \phi = \frac{a^2(a_1'^2 - b^2)}{a_1'^2(a^2 - b^2)},$$

$$\sin^2 \lambda = \frac{b^2(a^2 - b_1'^2)}{b_1'^2(a^2 - b^2)}, \quad \cos^2 \lambda = \frac{a^2(b_1'^2 - b^2)}{b_1'^2(a^2 - b^2)}.$$

$$\therefore CM = b_1 \cos \lambda = a \sqrt{\frac{b_1'^2 - b^2}{a^2 - b^2}}, \quad CN = a_1 \cos \phi = a \sqrt{\frac{a_1'^2 - b^2}{a^2 - b^2}},$$

$$PN = a_1 \sin \phi = b \sqrt{\frac{a^2 - a_1'^2}{a^2 - b^2}}, \quad DM = b_1 \sin \lambda = b \sqrt{\frac{a^2 - b_1'^2}{a^2 - b^2}}.$$

$$\therefore \frac{CM}{PN} = \frac{a}{b} \sqrt{\frac{b_1'^2 - b^2}{a^2 - a_1'^2}} = \frac{a}{b}, \text{ since } a_1'^2 + b_1'^2 = a^2 + b^2, \text{ and}$$

$$\frac{CN}{DM} = \frac{a}{b} \sqrt{\frac{a_1'^2 - b^2}{a^2 - b_1'^2}} = \frac{a}{b}.$$

The coördinates of the point P are $\left(a \sqrt{\frac{a_1'^2 - b^2}{a^2 - b^2}}, b \sqrt{\frac{a^2 - a_1'^2}{a^2 - b^2}}\right)$, and of A ,

$(a, 0)$; those of B are $(0, b)$, and of D' $\left(a \sqrt{\frac{b_1'^2 - b^2}{a^2 - b^2}}, -b \sqrt{\frac{a^2 - b_1'^2}{a^2 - b^2}}\right)$; therefore

$$\text{slope of } AP = b \sqrt{\frac{a^2 - a_1'^2}{a^2 - b^2}} \div a \left(\sqrt{\frac{a_1'^2 - b^2}{a^2 - b^2}} - 1 \right) = -\frac{b}{a} \frac{\sqrt{(a_1'^2 - b^2)} + \sqrt{(a^2 - b^2)}}{\sqrt{(a^2 - a_1'^2)}}.$$

$$\text{Slope of } BD' = -\frac{b}{a} \cdot \frac{1 + \sqrt{\left(\frac{a^2 - b_1'^2}{a^2 - b^2}\right)}}{\sqrt{\left(\frac{b_1'^2 - b^2}{a^2 - b^2}\right)}} = -\frac{b}{a} \cdot \frac{\sqrt{(a^2 - b_1'^2)} + \sqrt{(a^2 - b^2)}}{\sqrt{(b_1'^2 - b^2)}}.$$

By the relation $a_1'^2 + b_1'^2 = a^2 + b^2$, both slopes are equal; therefore PA is parallel to BD' . In the same way, AP' is parallel to BD .

II. Solution by G. W. GREENWOOD, M. A. (Oxon), Lebanon, Ill.

Since P and D are extremities of conjugate diameters, their coördinates may be written $(a\cos\phi, b\sin\phi)$, $(a\sin\phi, -b\cos\phi)$ where a and b are the semi-axes. We obtain the required result at once by disregarding the sign and considering only the absolute magnitudes of the coördinates. Equations to AP and BD are

$$\frac{x - a\cos\phi}{a - a\cos\phi} = \frac{y - b\sin\phi}{-b\sin\phi}, \text{ and } \frac{x - a\sin\phi}{-a\sin\phi} = \frac{y + b\cos\phi}{b + b\cos\phi},$$

and the inclinations are therefore

$$-\frac{b\sin\phi}{a(1 - \cos\phi)} \text{ and } -\frac{b(1 + \cos\phi)}{a\sin\phi},$$

which are easily seen to be equal.

Also solved by G. B. M. Zerr.

CALCULUS.

187. Proposed by L. T. JACKSON, St. Louis, Mo.

Find the area of the ellipse

$$\begin{aligned} x &= a_1 + a_2 \cos\theta + a_3 \sin\theta, \\ y &= b_1 + b_2 \cos\theta + b_3 \sin\theta. \end{aligned}$$

I. Solution by H. B. LEONARD, B. S., Chicago, Ill.

Let $x' = x - a_1$, $y' = y - b_1$; then eliminating $\cos\theta$ and $\sin\theta$ in turn we obtain

$$b_2 x' - a_2 y' = \Delta \sin\theta, \quad b_3 x' - a_3 y' = -\Delta \cos\theta,$$

where $\Delta = b_2 a_3 - a_2 b_3$. The substitution $x'' = b_2 x' - a_2 y'$, $y'' = b_3 x' - a_3 y'$, which multiplies areas by Δ , transforms the ellipse into the circle $x'' = \Delta \sin\theta$, $y'' = -\Delta \cos\theta$; i. e. $x''^2 + y''^2 = \Delta^2$.

The area of circle being $\pi \Delta^2$, the area of ellipse is $\pi \Delta = \pi(b_2 a_3 - a_2 b_3)$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Transform the origin to the center by writing x for $x - a_1$, y for $y - b_1$.

$$\therefore x = a_2 \cos\theta + a_3 \sin\theta, \quad y = b_2 \cos\theta + b_3 \sin\theta.$$

$$\begin{aligned} \text{Area} &= \int y dx = \int_0^{2\pi} (b_2 \cos\theta + b_3 \sin\theta)(a_3 \cos\theta - a_2 \sin\theta) d\theta \\ &= \int_0^{2\pi} (a_3 b_2 \cos^2\theta - a_2 b_3 \sin^2\theta - a_2 b_2 \sin\theta \cos\theta \pm a_3 b_3 \sin\theta \cos\theta) d\theta \\ &= \pi(a_3 b_2 - a_2 b_3). \end{aligned}$$

Also solved by F. P. Matz, and the Proposer.